The use of fuzzy relations in the assessment of information resources producers' performance

Marek Gagolewski[1,2] **Jan Lasek**[3]

[1]Systems Research Institute, Polish Academy of Sciences,
ul. Newelska 6, 01-447 Warsaw, Poland, gagolews@ibspan.waw.pl

[2]Faculty of Mathematics and Information Science, Warsaw University of Technology,
ul. Koszykowa 75, 00-662 Warsaw, Poland

[3]Interdisciplinary PhD Studies Program,
Systems Research Institute, Polish Academy of Sciences
j.lasek@phd.ipipan.waw.pl

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Introduction and motivation

In today's world with massive amounts of data we suffer from so-called *information overload*. There is a need for methods for *selection of valuable producers*.

The goals of our study are twofold:

- to overcome some of the drawbacks of standard approaches for evaluation of producers' outcomes and
- to provide tools for selecting interesting "producers".

Producer Assessment Problem (PAP)

Formal definition of the problem under our consideration [Gagolewski and Grzegorzewski 2011].

Producer Assessment Problem

Let $P = \{p_1, \ldots, p_k\}$ be a finite set consisting of k producers. The i-th producer outputs n_i products. Additionally, each product is given some kind of quantitative rating, e.g. concerning its overall quality.

The state of p_i may be described by a sequence

$$\mathbf{x}^{(i)} = \left(x_1^{(i)}, \ldots, x_{n_i}^{(i)}\right) \in \mathbb{I}^{1,2,\ldots} = \bigcup_{n \geqslant 1} \mathbb{I}^n$$

with elements in $\mathbb{I}$, e.g. $\mathbb{I} = [0, \infty)$. Most importantly, we should note that the numbers of products may vary from producer to producer.

Standard approach toward PAP (1)

The standard approach toward PAP is the use of so-called *impact functions* [Gagolewski and Grzegorzewski 2011, Gagolewski 2013, Quesada 2010, Woeginger 2008].

Definition (Impact function)

Impact function $\mathsf{F} : \mathbb{I}^{1,2,\dots} \to \mathbb{I}$ is a function with the properties that it is

- non-decreasing in each variable,
- arity-monotonic (additional product unit(s) does not reduce the overall producer evaluation),
- symmetric.

Standard approach toward PAP (2)

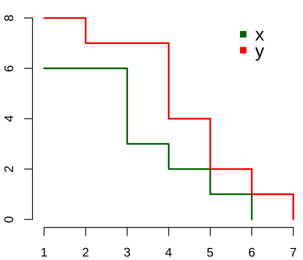
With the class of impact functions the following relation is connected.

Definition (Producers' dominance relation)

Let for $\mathbf{x} \in \mathbb{I}^n, \mathbf{y} \in \mathbb{I}^m$ a relation $\trianglelefteq \subseteq \mathbb{I}^{1,2,\dots} \times \mathbb{I}^{1,2,\dots}$ be defined as

$$\mathbf{x} \trianglelefteq \mathbf{y} \textit{ iff } n \leqslant m \text{ and } x_{(i)} \leqslant y_{(i)} \text{ for all } i \leqslant n,$$

where $x_{(i)}$ denotes the i-th greatest coordinate of vector $\mathbf{x}$.



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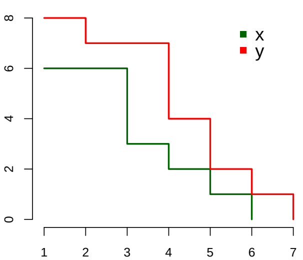

In fact, the following theorem applies [Gagolewski and Grzegorzewski 2011]:

Theorem

Let $\mathsf{F} : \mathbb{I}^{1,2,\dots} \to \mathbb{I}$ *be an aggregation operator. Then* F *is symmetric, nondecreasing with respect to each variable and arity-monotonic if and only if for any* $\mathbf{x}, \mathbf{y} \in \mathbb{I}^{1,2,\dots}$ *if* $\mathbf{x} \trianglelefteq \mathbf{y}$, *then* $\mathsf{F}(\mathbf{x}) \leqslant \mathsf{F}(\mathbf{y})$.

Issues with the standard approach toward PAP

There are several issues that come with the standard approaches toward PAP:

- Embedding of producers' output into, e.g., $\mathbb{I} = [0, \infty)$, where linear order induced by $\leqslant$ is present, gives the comparison between producers *out of control*.

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- If one wants to assign *equal scores* to incomparable cases w.r.t. $\trianglelefteq$ then the impact function needs to be *trivial*, i.e, $\forall \mathbf{x}\ \mathrm{F}(\mathbf{x}) = c$ for some $c \in \mathbb{I}$ [Gagolewski 2013, Theorem 3].

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- One can arrive at *any desired ranking* of producers by an *appropriate construction* of an impact function [Gagolewski 2013, Theorem 4].

Fuzzy approach toward PAP

First, we recall the definition of a fuzzy relation.

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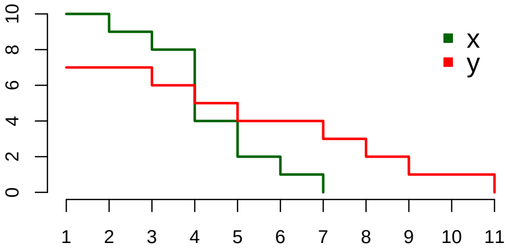
These notions have their natural counterparts in "crisp" setting.

Exemplary class of fuzzy preference relation for PAP (1)



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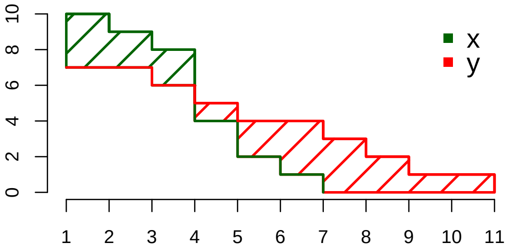

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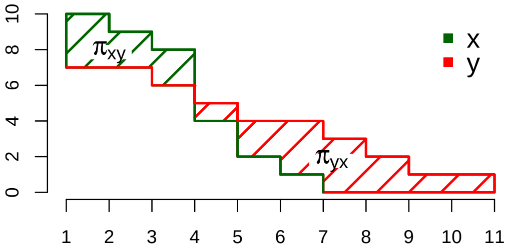

Exemplary class of fuzzy preference relation for PAP (2)

$\mathcal{S}$:= space of infinite nonincreasing sequences with elements in $\mathbb{I}$.
Let $\tilde{\cdot} : \mathbb{I}^{1,2,\ldots} \to \mathcal{S}$ be an operator such that for $\mathbf{x} \in \mathbb{I}^n$ we have $\tilde{\mathbf{x}} = (x_{(n)}, x_{(n-1)}, \ldots, x_{(1)}, 0, 0, \ldots)$.

Definition (Fuzzy producers' dominance relation)

Let $\mathbf{x}, \mathbf{y} \in \mathcal{S}$, and $\mathbf{w} = (w_1, w_2, \ldots)$, $w_i > 0$ for all i. The *fuzzy producers dominance relation* is a fuzzy preference relation $\blacktriangleleft$ with the membership function given by:

$$\mu(\mathbf{x}, \mathbf{y}) = \begin{cases} \frac{\pi_{yx}}{\pi_{xy} + \pi_{yx}} & \text{if } \pi_{xy} + \pi_{yx} > 0, \\ 0.5 & \text{otherwise,} \end{cases}$$

where $\pi_{xy} = \sum_i w_i \cdot \max\{x_i - y_i, 0\}$.

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Hence, the relation is *fuzzy preference relation* in the sense studied by, e.g. Tanino [1988].

Aggregation of pairwise comparisons to a ranking

Producers' scores may be derived by e.g. the net flow method [Bouyssou 1992, Fodor and Roubens 1994]. This method assigns scores according to the formula

$$S_{\text{net}}(\mathbf{x}^i) = \sum_{\mathbf{x}^j \in \mathcal{X}} \mu(\mathbf{x}^j, \mathbf{x}^i) - \mu(\mathbf{x}^i, \mathbf{x}^j) = \text{“}\textit{inflow}\text{”} - \text{“}\textit{outflow}\text{”}$$

Producers are ranked with respect to their scores. This is quite analogous to the classical approach in which the impact functions are used.

Quality of rankings

We also would like to suggest an evaluation (quality) measure Q for ranking r. We require that the measure has at least the following properties:

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The following function can constitute an exemplary quality measure:

$$Q(r, \blacktriangleleft) = \frac{\sum\limits_{\substack{i,j:\\ r(\mathbf{x}^i) > r(\mathbf{x}^j)}} \mu(\mathbf{x}^i, \mathbf{x}^j) + \sum\limits_{\substack{i<j:\\ r(\mathbf{x}^i) = r(\mathbf{x}^j)}} 1 - 2\left|\mu(\mathbf{x}^i, \mathbf{x}^j) - \frac{1}{2}\right|}{\binom{n}{2}}.$$

Application - Ranking of users at StackOverflow (1)

stackoverflow | Questions | Tags | Users | Badges | Unanswered

Why is this program erroneously rejected by three C++ compilers?

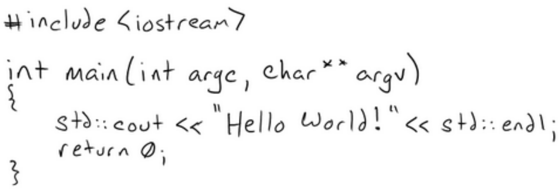
474

873

I am having some difficulty compiling a C++ program that I've written.

This program is very simple and, to the best of my knowledge, conforms to all the rules set forth in the C++ Standard. I've read over the entirety of ISO/IEC 14882:2003 twice to be sure.

The program is as follows:



Here is the output I received when trying to compile this program with Visual C++ 2010:

```
c:\dev>cl /nologo helloworld.png
cl : Command line warning D9024 : unrecognized source file type 'helloworld.png', objec
helloworld.png : fatal error LNK1107: invalid or corrupt file: cannot read at 0x5172
```

Application - Ranking of users at StackOverflow (2)

We applied chosen methods to rank 100 most active users on StackOverflow (with the biggest number of answers).

We confronted our results with standard approaches using

- StackOverflow reputation index i_R,
- average quality of an answer $\bar{\mathbf{x}}$,
- maximal quality answer $x_{(n)}$,
- sum of quality over answers $\Sigma(\mathbf{x})$,
- number of answers n,
- Hirsch's h-index i_H and Woeninger w-index i_W,
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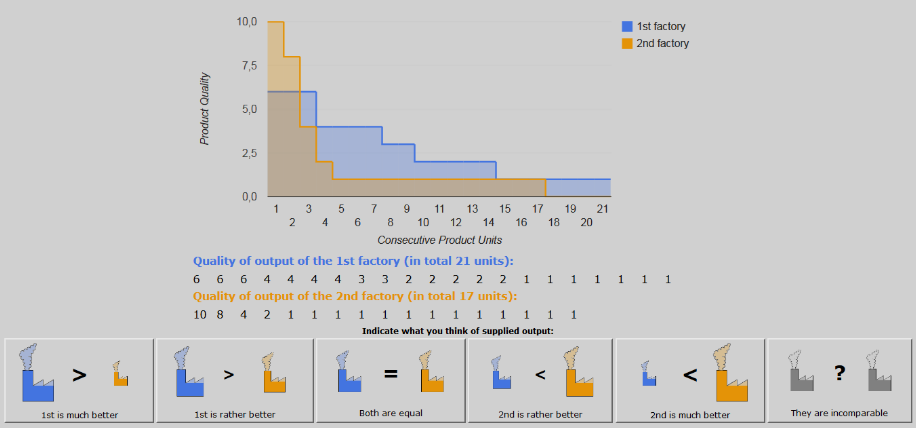
Table: Quality measures of rankings.

i_R	$\bar{\mathbf{x}}$	$\mathbf{x}_{(n)}$	$\Sigma(\mathbf{x})$	n	i_H	i_W	NF	SO
0.895	0.748	0.749	**0.88**	0.726	0.831	0.819	**0.874**	**0.914**

Future work - learning relations from data: questionnaire



Summary and future work

- In our study we employed tools from fuzzy logic and fuzzy set theory to Producer Assessment Problem. We argue that it is a more gentle approach for evaluation of producers.
- The fuzzy pairwise comparison relation allows us to naturally extend its counterpart in the crisp setting in which many pairs of producers are incomparable.
- The proposed relation founds a basis of the ranking and is the most important ingredient of the model. Such a relation might be constructed not only by an explicit formula, but by statistical or machine learning models.

Thank you!

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