Football team rankings

Jan Lasek

Overview

- Why rankings are important
- Different approaches towards ratings
- Comparison between models

Importance of rankings

- Useful in scheduling even match-ups
- Seedings for the competitions
- According to the official FIFA ranking position work permit for players are granted by UK government (2006)

The official FIFA ranking

- Current method introduced after WC 2006
- Teams are awarded points accoring to the formula:

$$Points = P \times M \times S \times C$$

Where
- **p**oints for match result (0, 1, 2, 3)
- **m**atch importance (1, 2.5 , 3 or 4)
- opposition **s**trength = max(200 – position, 50)
- average **c**onfederation strength [0.85, 1]

Example calculation

Netherlands 2:1 Brazil
in World Cup Quarterfinals

Points for Netherlands: $3 \times 4 \times 200 \times 1 = 2400$

Points for Brazil: $\qquad 0 \times 4 \times 196 \times 1 = 0$

- In each of last 4 years of play average number of points are calculated.
- Averages are summed up with weights 0.2, 0.3, 0.5, 1 for consecutive years

The official FIFA ranking (2)

- In each of last 4 years of play average number of points are calculated.

- Averages are summed up with weights 0.2, 0.3, 0.5, 1 for consecutive years

- In addition, less active teams (with less than 5 games played in a year) are penalized by points division.

What chess has to do with football?

- Elo rating system

Elo rating system (1)

- Eloratings.net
- Official FIFA Women World Rankings
- Simple update rule:

$$R_{new} = R_{old} + K(Score - Expected)$$

- Calculation of expected result - example

Let us consider two teams in the ranking: Poland (690) and Greece (745).

Differece in ratings is equal to -65. We assume that a team playing at home ground receive 100 points extra. Then Poland is rated with 35 points more than Greece.

Elo rating system (2)

Expected result from prespective of Polish team becomes

$$P(Poland\ wins) = \frac{1}{1 + 10^{-35/400}} \approx 0.55$$

Poland (hopefully) wins 2:1. Greece lose but with small margin – this result is mapped as 0.15 for them and as 1 - 0.15 = 0.85 for Poland.

New ratings become

$$R_{Poland} = 690 + 15 \cdot 3(0.85 - 0.55) = 703.5$$
$$R_{Greece} = 745 + 15 \cdot 3(0.15 - 0.45) = 731.5$$

Improvement to Elo system

- Elo++ model – Kaggle.com competition on chess players ratings winning solution

Find ratings r_i that minimize error function

$$\sum_{games} w(s-p)^2 + \lambda \sum_{teams} (r-a)^2$$

w — *time weight*

s *and* p — *actural and predicted score*

a — *average rating of opponents*

Elo +- ratings

Team
1.   New Caledonia
2.   Spain
3.   Brazil
4.   Netherlands
5.   Solomon Islands
6.   Argentina
7.   Germany
8.   Chile
9.   Portugal
10. England

Which sport is it?

Least squares ratings

- We assume that goal difference y_{ij} is proportional to difference in strenghts (ratings):

$$y_{ij} = r_i - r_j + \varepsilon$$

- We may also make correction for advantage of home team:

$$y_{ij} = r_i + h - r_j + \varepsilon$$

- Impose constraint to find model parameters:

$$r_1 + r_2 + \cdots + r_n = 0$$

Least squares ratings

	Team	Rating
1.	Spain	3.713
2.	Brazil	3.429
3.	Netherlands	3.274
4.	Germany	3.243
5.	England	3.055
6.	Uruguay	3.040
7.	Argentina	2.920
8.	Portugal	2.904
9.	Mexico	2.704
10.	Chile	2.631

If you work out the formula for ratings you arrive with

$$r_i = \frac{gd_i + \sum_{i \neq j} r_j g_{ij}}{G_i}$$

Social network ratings (1)

- Primairly used to determine relative importance of an actor by counting his/her aquintances

- Based on an intuitive principle – fans argue which team is better. Their teams have not played each other, but both of them played a third team – one of them lost, the second won
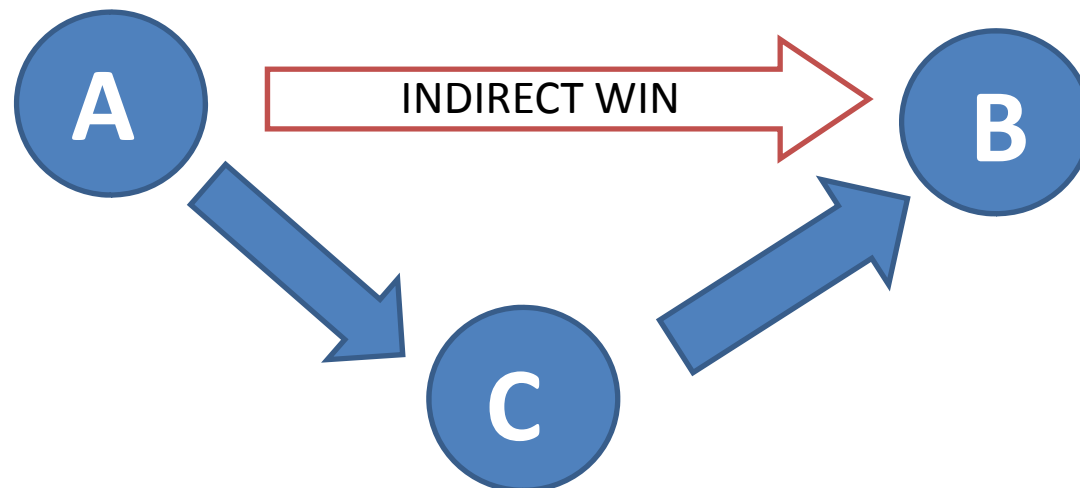
Social network ratings (2)

- The supporter of team B claims that therefore his team is the better one
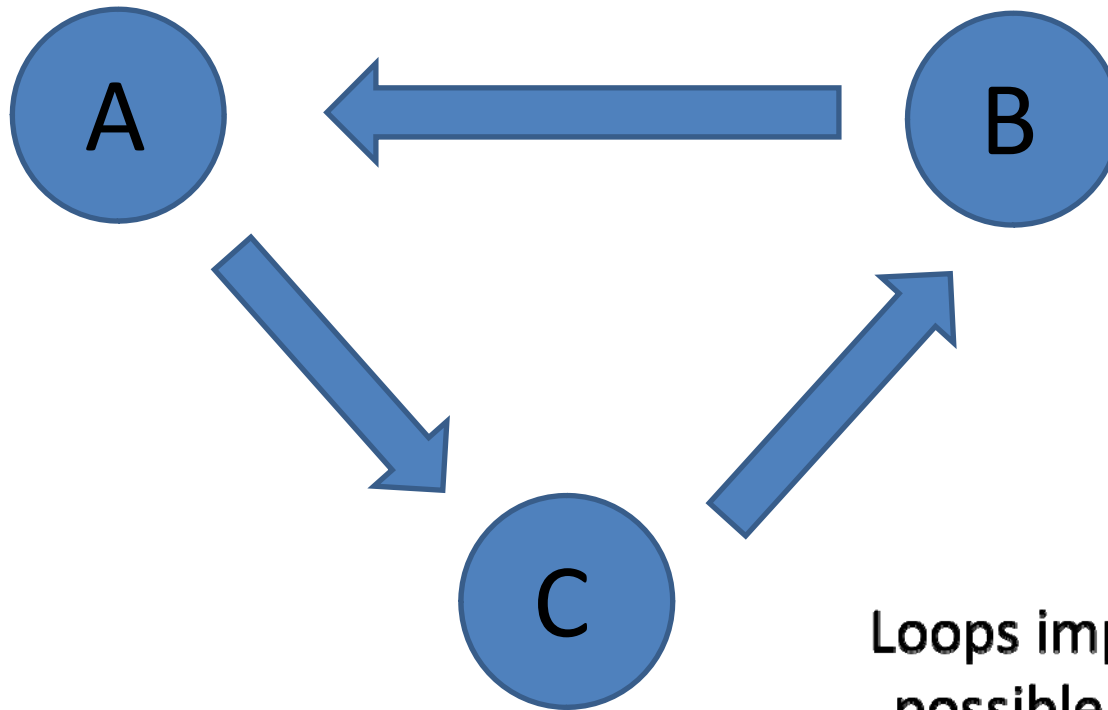
We calculate all direct and indirect wins/losses for every team. We also discount the victory/loss by

$$\alpha^{k-1}$$

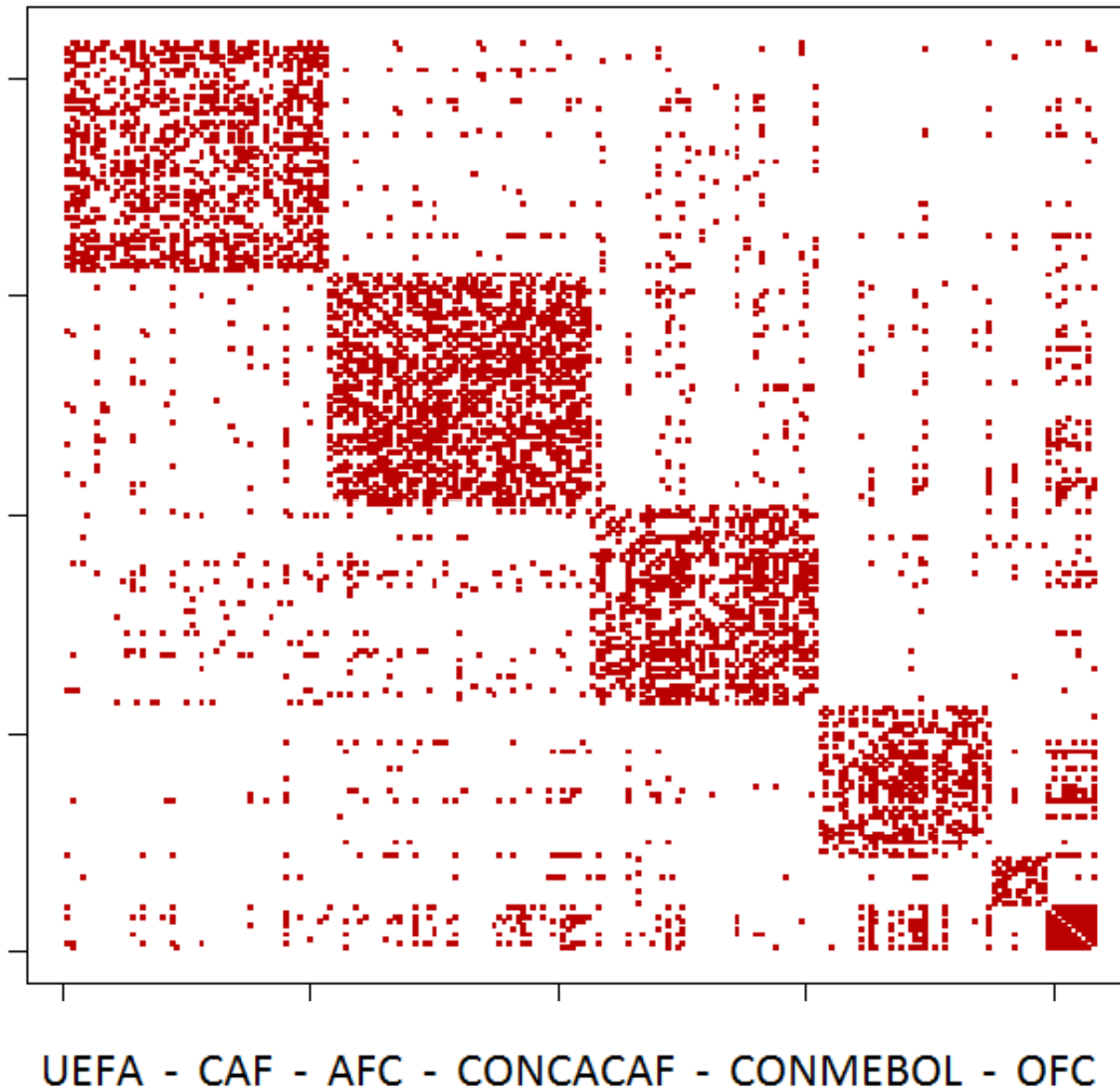when it corresponds to a path of lenght k. Set ratings to:

$$r_i = w_i - l_i$$

α should be in unit interval!

Loops in the network

Loops impose restrictions on possible values of discount parameter α

$$1 + A + A^2 + A^3 + \cdots$$

Schedule – adjacency matrix

UEFA - CAF - AFC - CONCACAF - CONMEBOL - OFC

Social network ratings (2)

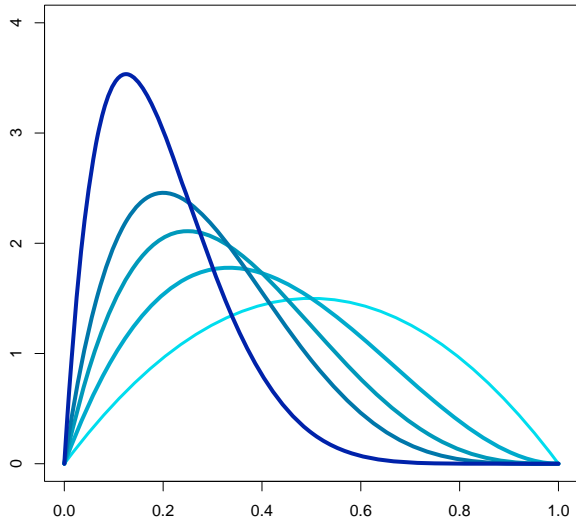| Pos	Team	Ratings
1.	Spain	70.25
2.	Netherlands	51.1
3.	Brazil	49.71
4.	Iran	45.51
5.	Egypt	43.90
6.	Germany	43.01
7.	Korea Republic	40.08
8.	Japan	39.45
9.	Côte d'Ivoire	37.21
10.	Argentina	34.93
11.	Australia	34.91
12.	Uruguay	32.56
13.	England	31.72
14.	Croatia	29.76
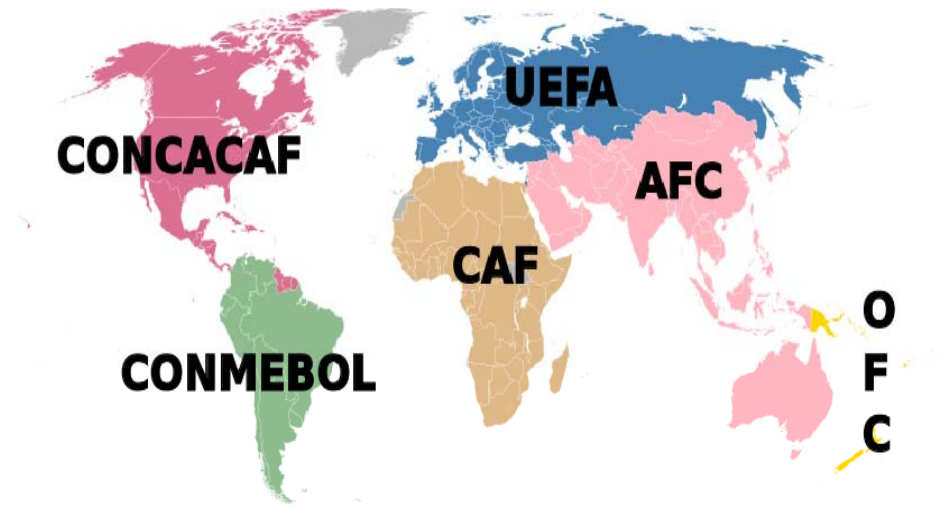
Markovian ratings (1)

- We consider „a glory supporter" who is cheering for winning teams/teams that score many goals

- In addition, the supporter is *memoryless*

- We may model his behaviour by a Markov chain

- Such idea is applied in *PageRank* algorithm for rating web pages

Head-to-head probabilites
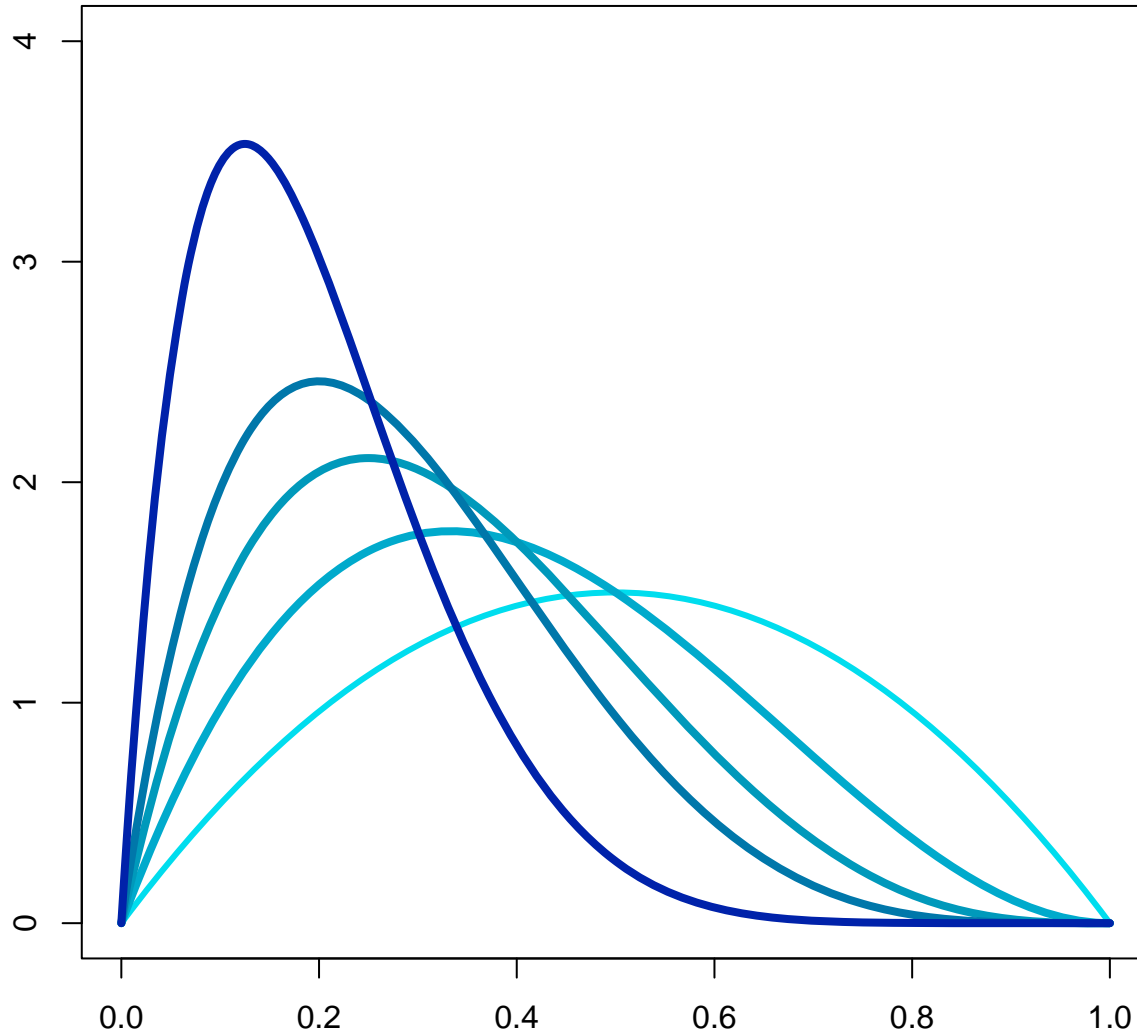
$$\frac{Wins_A + 1}{Wins_A + Wins_B + 2}$$

$$\frac{Goals_A + 1}{Goals_A + Goals_B + 2}$$

1	Spain	2.518
2	Brazil	2.19
3	Germany	1.904
4	Netherlands	1.893
5	Mexico	1.625
6	Argentina	1.619
7	Uruguay	1.532
8	Japan	1.516
9	France	1.442
10	Korea Republic	1.43
11	Portugal	1.427
12	England	1.401
13	South Africa	1.32
15	Chile	1.305
15	Paraguay	1.29
16	Italy	1.26
17	Poland	1.255
18	Côte d'Ivoire	1.254

Comparison between the models (1)

Ranking	Time factor	Goal scored	Match importance	Home team advantage
FIFA ranking	✓	✗	✓	✗
Elo WWR	✗	✓	✓	✓
Elo ratings.net	✗	✓	✓	✓
Elo ++	✓	✗	✗	✓
Least squares	✗	✓	✗	✓/✗
Network based system	✗	✗	✗	✗
Markovian ratings	✗	✓/✗	✗	✗

Comparison between the models

- Prediction function as in Elo model

$$P(Team\ i\ wins) = \frac{1}{1 + e^{-a(r_i - r_j) - h}}$$

- Evaluation is based on 979 games played between 1st April 2011 and 2nd May 2012
- We use daily updated rankings

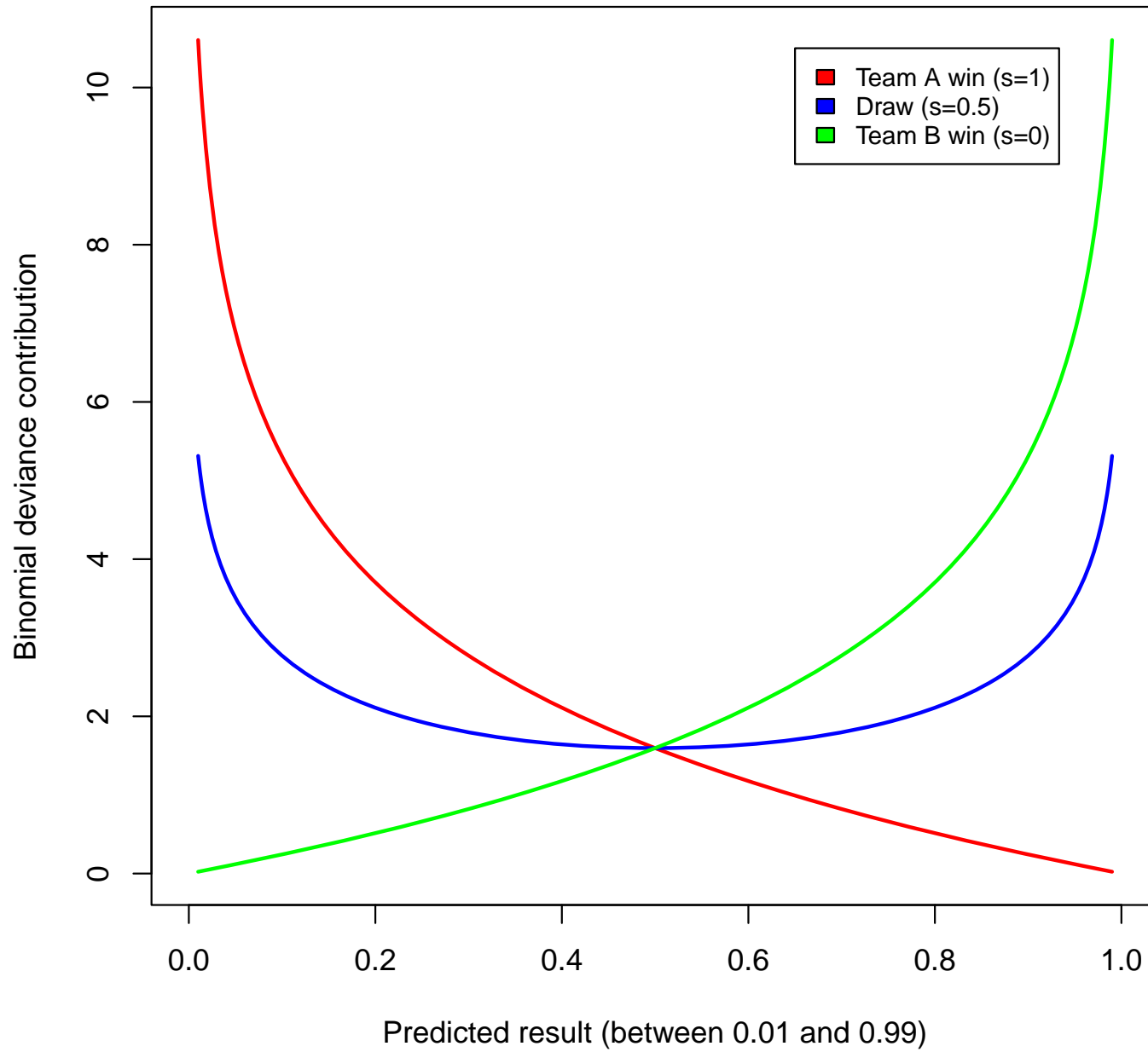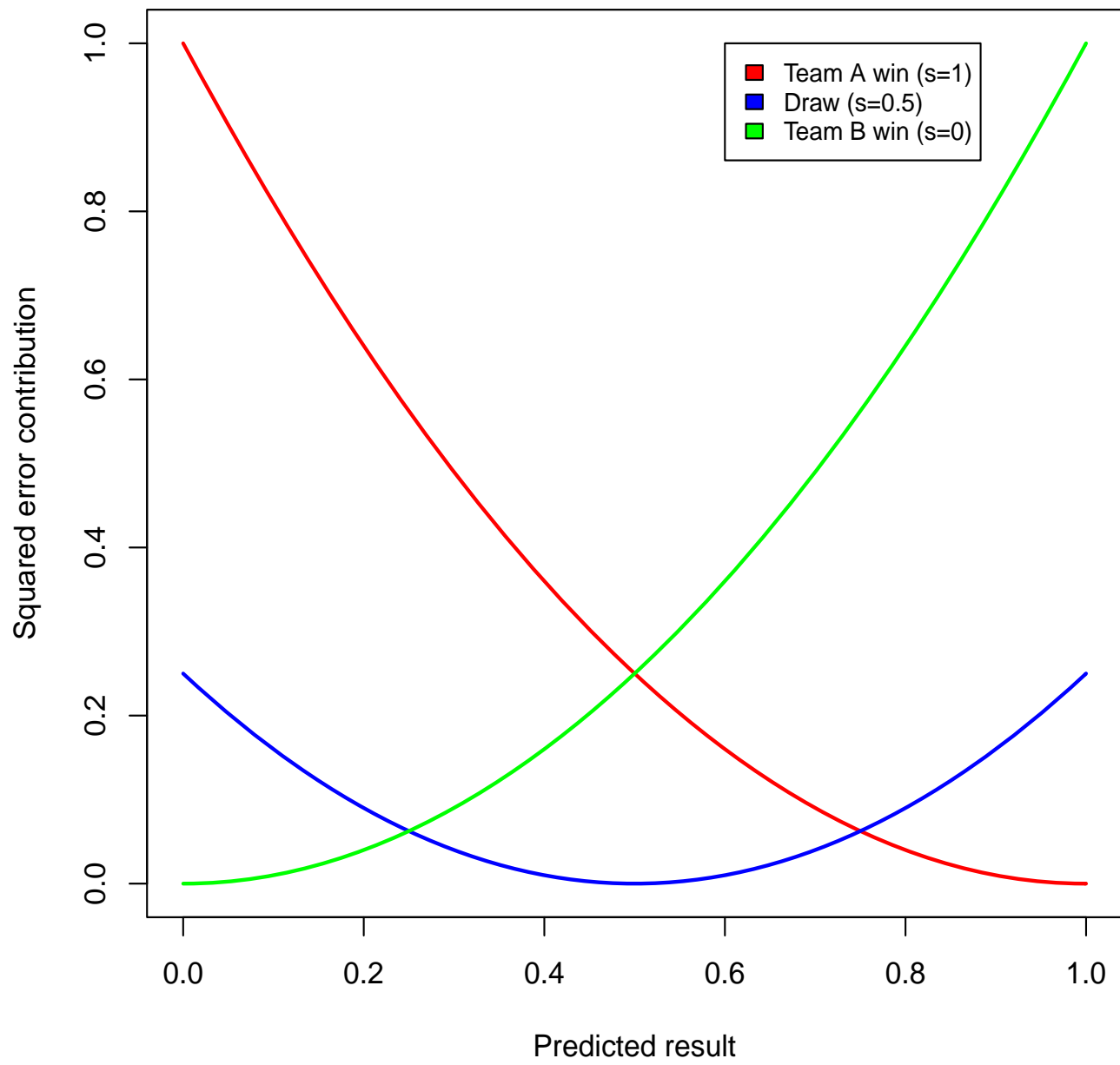|	
Friendlies	**461**
FIFA WC Qualifier	**219**
Continental Qualifier	**219**
Contiental Final	**80**

Evaluation measures

Binomial deviance (log likelihood)

$$-[s_i \log(p_i) + (1 - s_i) log(1 - p_i)]$$

Mean squared error

$$(s_i - p_i)^2$$

Ranking	BinDev	90% conf. interval	MSE	90% conf. interval
FIFA monthly	1.3705	(1.3504, 1.3905)	0.1450	(0.125, 0.1651)
FIFA daily	1.3681	(1.3481, 1.3880)	0.1443	(0.1244, 0.1643)
Elo WWR 1500	1.3698	(1.3498, 1.3898)	0.1447	(0.1246, 0.1647)
Elo WWR FIFA06	**1.2674**	**(1.2489, 1.2861)**	**0.1268**	**(0.1081, 0.1455)**
Elo WWR FIFA06 01	1.2934	(1.2744, 1.3123)	0.1302	(0.1113, 0.1492)
Elo ratings.net 1500	1.3265	(1.3070, 1.346)	0.1370	(0.1176, 0.1565)
Elo ratings.net FIFA06	1.2811	(1.2624, 1.2999)	0.1280	(0.1092, 0.1468)
Elo ratings.net	**1.2634**	**(1.2446, 1.2821)**	**0.1271**	**(0.1084, 0.1458)**
Elo++ (λ, h) = (0.05, 0.5)	1.288	(1.2690, 1.3069)	0.1305	(0.1115, 0.1494)
Least squares	1.2786	(1.2597, 1.2975)	0.1288	(0.11, 0.1477)
Least squares *HTA*	**1.2681**	**(1.2493, 1.2869)**	**0.1272**	**(0.1085, 0.146)**
Network ratings	1.4224	(1.4018, 1.4431)	0.1540	(0.1333, 0.1746)
Network ratings *months*	1.3499	(1.3303, 1.3696)	0.1398	(0.1201, 0.1595)
PowerRank.com	1.2862	(1.2672, 1.3053)	0.1305	(0.1115, 0.1496)
Markov *Wins*	1.3588	(1.3391, 1.3786)	0.1406	(0.1209, 0.1604)
Markov *Goals*	1.3541	(1.3344, 1.3738)	0.1399	(0.1202, 0.1595)
Draws	1.5960		0.1902	