Learning experts’ preferences from informetric data

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IFSA-EUSFLAT
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Introduction and motivation

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Introduction and motivation

Lotfi A. Zadeh
Professor Emeritus, EECS, UC Berkeley
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Decision-making in a fuzzy environment
Introduction and motivation

The field of *informetrics* deals with measurable aspects of information processes. So far, a number of tools has been suggested to quantify the value of information.

In this exposition we will investigate the efficacy of a set of chosen off-the-shelf solutions in an exemplary setup.
Producer Assessment Problem (PAP)

Let us formally define the problem under our consideration [Gagolewski and Grzegorzewski 2011].

Producer Assessment Problem

Let \( P = \{p_1, \ldots, p_k\} \) be a finite set consisting of \( k \) producers. The \( i \)-th producer outputs \( n_i \) products. Additionally, each product is given some kind of quantitative rating, e.g. concerning its overall quality. The state of \( p_i \) may be described by a sequence

\[
x^{(i)} = (x_1^{(i)}, \ldots, x_{n_i}^{(i)}) \in I_1 \times I_2 \times \ldots = \bigcup_{n \geq 1} I^n
\]

with elements in \( I \), e.g. \( I = [0, \infty) \). Most importantly, we should note that the numbers of products may vary from producer to producer. The goal is to design tools for producers’ evaluation (rankings, preference relations, etc.) and their impact measurement.
Figure: Illustration of PAP definition for two example output vectors $x = (10, 9, 8, 4, 2, 1)$ and $y = (7, 7, 6, 5, 4, 4, 3, 2, 1, 1)$. 
Available tools for analysis (1)

Up until today, a number of tools were proposed for Producer Assessment Problem including:

Hirsch's $h$-index $h = \max \{ i : x_i - i \}$ [Hirsch 2005],
Egghe's $g$-index $g = \max \{ i : \sum_{j=1}^{i} x_i - i^2 \}$ [Egghe 2010],
Woeinger's $w$-index $w = \max \{ i : x_j - i - j + 1 \text{ for all } j = 1, \ldots, i \}$ [Woeinger 2008].

These are examples of so-called impact indexes.
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These are examples of so-called **impact indexes**.
Available tools for analysis (2)

There are also tools from the domain of fuzzy systems. For example, the following fuzzy preference relation was suggested [Gagolewski and Lasek 2015]. For two output vectors $\mathbf{x}$ and $\mathbf{y}$, the membership function of fuzzy preference relation $\mathbf{x} \preceq \mathbf{y}$ is given by

$$
\mu(\mathbf{x}, \mathbf{y}) = \begin{cases} 
\frac{\pi_{yx}}{\pi_{xy} + \pi_{yx}} & \text{if } \pi_{xy} + \pi_{yx} > 0, \\
0.5 & \text{otherwise},
\end{cases}
$$

where $\pi_{xy}$ and $\pi_{yx}$ are membership functions.

![Diagram showing membership functions $\pi_{xy}$ and $\pi_{yx}$]
Research question

In this exposition, our research question is **which of the proposed functions (if any) is effective in describing experts’ preferences in an exemplary instance of Producer Assessment Problem. In other words:**

¿Do these tools effectively compress information contained in data?

We prepared generated data for PAP for an on–line questionnaire. The participants’ (experts’) responses serve us as evidence for validation purposes.
Questionnaire

In the questionnaire, participants were asked to provide answers for a series of questions.

![Graph showing product quality over consecutive product units for two factories.]

**Quality of output of the 1st factory (in total 21 units):**
6 6 6 4 4 4 4 3 3 2 2 2 2 2 1 1 1 1 1 1 1

**Quality of output of the 2nd factory (in total 19 units):**
13 8 4 3 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1
Validation of hypothesis

To validate the hypothesis we confront two approaches:

- compare vectors on each coordinate and equalize their lengths by padding the shorter ones with zeros

\[(x_1, x_2, \ldots, x_n) \rightarrow (x_1, x_2, \ldots, x_n, 0, 0, \ldots, 0)\]

- extract certain features of output vectors using the discussed tools

\[(x_1, x_2, \ldots, x_n) \rightarrow (f_1(x), f_2(x), \ldots, f_k(x))\]

We use several prediction models:

- Ordinal Logistic Regression,
- $k$-Nearest Neighbours classifier and
- Random Forest model.

The models are trained on 80% of data and evaluated on 20% ($\approx 1000$ instances). In consecutive slides we discuss evaluation metrics used.
Evaluation metrics (1)

For $i$th example in the data set, $i = 1, 2, \ldots, N$ let

- $l_{t}^{(i)}$, $t \in \{-2, -1, 0, 1, 2\}$ denote true preference label,
- a given model assign probability $\mathbb{P}(l_{k}^{(i)})$ to label $l_{k}$,
- a classifier assign labels according to $\hat{l}_{p}^{(i)} = \arg\max_{k} \mathbb{P}(l_{k}^{(i)})$.

We considered the following evaluation measures:

- Missclassification rate

$$
\text{Misscl} = \frac{1}{N} \sum_{i=1}^{N} \mathbb{I}(l_{t}^{(i)} \neq \hat{l}_{p}^{(i)}).
$$

- average distance between labels for $d(l_{t}^{(i)}, l_{p}^{(i)}) = |t - p|$

$$
\text{AvgDist} = \frac{1}{N} \sum_{i=1}^{N} d(l_{t}^{(i)}, \hat{l}_{p}^{(i)}).
$$
Evaluation metrics (2)

- **Rank Probability Score**
  
  \[
  RPS = \frac{1}{4N} \sum_{i=1}^{N} \sum_{j=-2}^{2} \left( \hat{F}^{(i)}(j) - F^{(i)}(j) \right)^2,
  \]

  with \( \hat{F}^{(i)}(\cdot) \) and \( F^{(i)}(\cdot) \) being observed and estimated cumulative distribution function for labels

- **Concordance Index**
  
  \[
  C = \frac{1}{M} \sum_{i: \ l_t^{(i)} \neq 0} \mathbb{1}(l_t^{(i)}, \hat{l}_p^{(i)} \text{ concordant}) + 0.5 \cdot \mathbb{1}(\hat{l}_p^{(i)} = 0).
  \]

  with \( M \) being the number of “usable pairs” (i.e., \( l_t^{(i)} \neq 0 \))
Results - evaluation of models

Below we present the results of experiment for the two approaches (marked with superscript \( i \) and \( c \) for the “index” and “coordinate” approach respectively).

Table: Results of classification.

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<tr>
<th></th>
<th>Misscl</th>
<th>AvgDist</th>
<th>RPS</th>
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<tr>
<td>OLR(_i)</td>
<td>0.409</td>
<td>0.465</td>
<td>0.086</td>
<td>0.08</td>
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<tr>
<td>OLR(_c)</td>
<td>0.394</td>
<td>0.454</td>
<td>0.082*</td>
<td>0.075</td>
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<tr>
<td>( k)NN(_i)</td>
<td>0.401*</td>
<td>0.457*</td>
<td>0.085*</td>
<td>0.083*</td>
</tr>
<tr>
<td>( k)NN(_c)</td>
<td>0.453</td>
<td>0.548</td>
<td>0.099</td>
<td>0.122</td>
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<tr>
<td>RF(_i)</td>
<td>0.385*</td>
<td>0.452*</td>
<td>0.076*</td>
<td>0.078</td>
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<tr>
<td>RF(_c)</td>
<td>0.434</td>
<td>0.537</td>
<td>0.094</td>
<td>0.092</td>
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<tr>
<td>Equal</td>
<td>0.865</td>
<td>1.255</td>
<td>0.202</td>
<td>0.5</td>
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</table>
Results - feature importance (1)

For OLR model and different versions of kNN model (for various values of parameter k) we calculated how many times a given feature was picked by the employed feature selection procedure for different respondents. In this way, we obtain that the most important for classification are:

1. $i_G$ (picked 42 times)
2. $\Sigma(x)$ (30)
3. $\bar{x}$ (27)
4. $x_1$ (16)
5. $FP$ (15)
In case of Random Forest model we derived ranking of features aggregating individual importance rankings for 32 participants by Borda count. The following ranking of features was obtained (top 5):

1. $FP$
2. $i_G$
3. $\Sigma(x)$
4. $\bar{x}$
5. $x_1$
Summary of results

Our findings can be summarized in the following points:

- The emphasis was put on quality rather than productivity during the evaluation process.
- The available tools are effective in compressing information from producers' output vectors.
- Among the best performing aggregation tools in our experiment we identified Egghe's $g$-index $G$, sum of product qualities $\sum(x)$, the fuzzy $p$ reference relation $FP$, mean quality of a product $\bar{x}$ and the maximal quality of a product $x_1$. 
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Thank you for your attention!